Chapter 28: Special Relativity

Essential Concepts and Summary

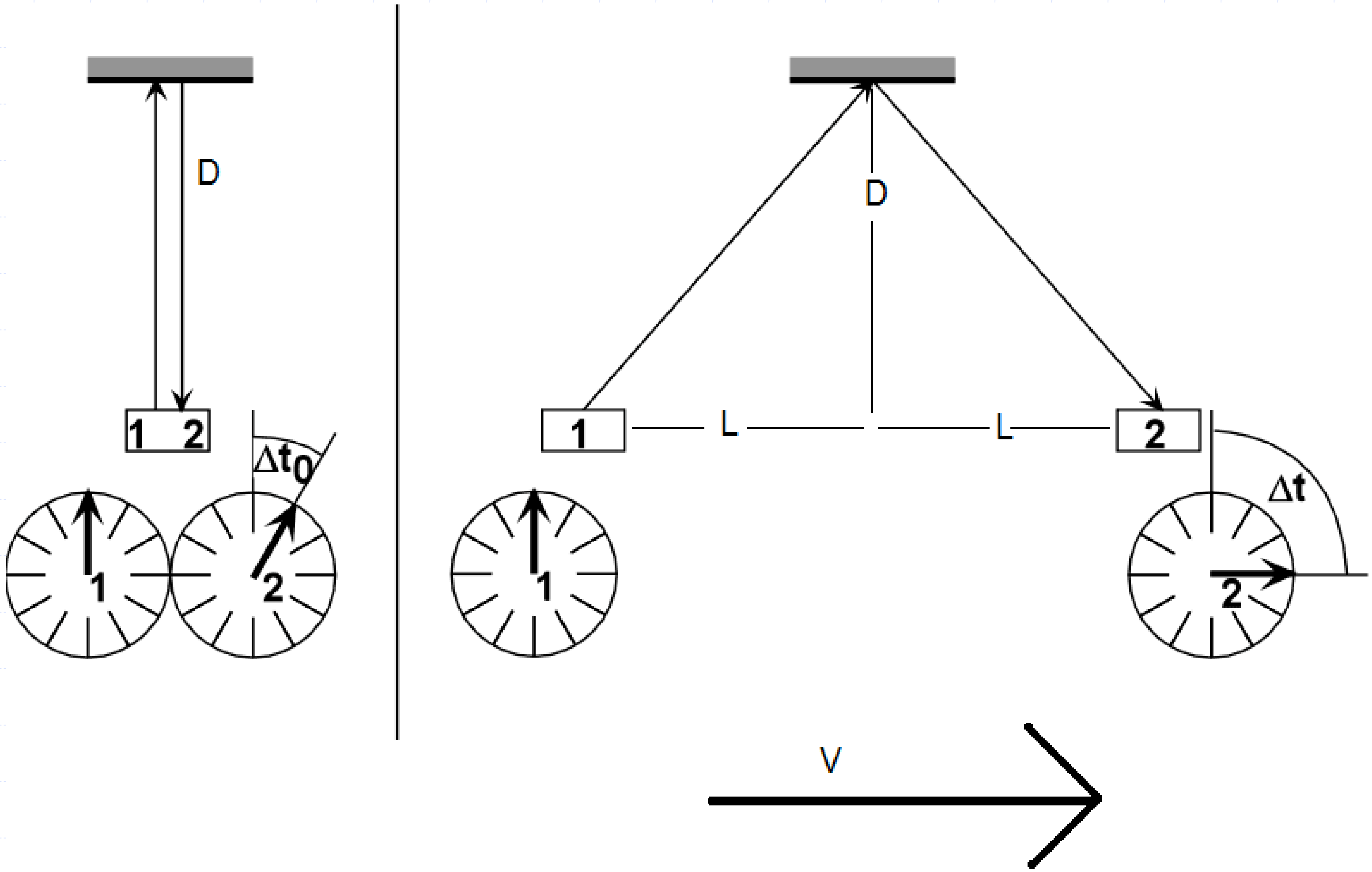
Events and Inertial Reference Frames

- Event: a physical "happening" that occurs at a certain place and time
- Reference frame: a set of x,y,z axes (coordinate system) and a clock
- Inertial reference frame: one in which Newton's law of inertia is valid.
- No such thing as "absolute" reference frame
- Any inertial frame is as good as any other for expressing laws of physics

Postulates of Special Relativity

- Postulate: fundamental assumption
- 1. Relativity Postulate: The laws of physics are the same in every inertial reference frame
- 2. Speed of Light Postulate: The speed of light in a vacuum, measured in any inertial reference frame, always has the value c, **regardless of the speed of observer and the source of light**.
- Called **special** because its applies only to the special case of frames of reference moving at a constant speed relative to each other

Time Dilation



Time Dilation

- Because speed of light is always constant, and speed is distance over time, some other constant has to change. In special relativity, a phenomenon called **time dilation** occurs.
- In the previous picture, we look at the astronaut from his own reference frame, then an outside, still inertial, reference frame

$$2L = v\Delta t$$

$$2s = 2\sqrt{D^2 + \left(\frac{v\Delta t}{2}\right)^2}$$

$$2s = c\Delta t$$

$$c\Delta t = 2\sqrt{D^2 + \left(\frac{v\Delta t}{2}\right)^2}$$

$$\Delta t = \frac{2D}{c} \cdot \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\frac{2D}{c} = \Delta t_0$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Length Contraction

- Because of time dilation, relativistic length is less than the **proper length** between two points
- Length contraction only occurs in the direction of motion

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

Mass Increase

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Relativistic Momentum

- Like speed and time, relativity modifies our ideas about momentum.
- Like normal momentum, however, relativistic momentum in an isolated system is conserved

$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Equivalence of Mass and Energy

- One of the most amazing results of special relativity is mass and energy are equivalent.
- Rest energy of an object is the special case when its velocity is 0

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$E_0 = m_0 c^2$$

$$KE = E - E_0$$

$$KE = mc^2 - m_0 c^2$$

$$= (m - m_0)c^2$$

Relativistic Addition of Velocities

- For a general situation, the relative velocities are related by the velocity addition formula.
- At speeds much below the speed of light, this is equivalent to our current understanding of addition of velocities
- V_{xy} is velocity of object X relative to object Y

$$v_{AB} = \frac{v_{AC} + v_{CB}}{1 + \frac{v_{AC} v_{CB}}{c^2}}$$

Equation Summary

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$v_{AB} = \frac{v_{AC} + v_{CB}}{1 + \frac{v_{AC}v_{CB}}{c^2}}$$

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$E_0 = m_0 c^2$$

$$KE = mc^2 - m_0 c^2$$